Strings and M-Theory

by Stephen Hawking

In the 1990's the subject formerly known as `string theory' evolved into something else, which has now become known as `M-theory.' M-theory is a circle of ideas connecting strings, quantum gravity, unification of forces, duality, Kaluza-Klein theory, Yang-Mills theory, and supersymmetry. While the fundamental principles of M-theory are still unclear, our picture of the subject has evolved rapidly in recent years. M-theory has the distinction of being the only approach to quantum gravity which has succeeded both in tying itself firmly to our classical understanding of gravity (albeit in 10 or 11 dimensions) and in addressing non-perturbative quantum issues such as the entropy of black holes. (See Classical and Quantum Gravity for other approaches to quantum gravity.) To some researchers M-theory is a candidate for a `theory of everything' which would underlie all of the structures in our universe. Whether or not this is the case, there is no doubt that M-theory is an active arena for the development of ideas in quantum gravity, cosmology, and field theory.

M-theory and the physics of p-branes

String theory used to be a theory of, well, strings. In the not so recent past one could hear string theorists state that the fundamental principle of string theory was that the things we think of as particles (electrons, photons, gravitons, etc.) are in reality extended objects that look like closed vibrating loops of string. All distinctions between the particles would derive from the association of each particle with a different normal mode of vibration.

The picture is now quite different. In addition to strings, M-theory contains a zoo of higher dimensional objects; e.g. 2-dimensional membranes (aka 2-branes), 3-dimensional `3-branes', etc. An object with p spatial dimensions is known as a p-brane. These branes are now thought to be as fundamental as the famous `fundamental string.' Indeed, the various branes are related to fundamental strings by powerful symmetries (known as dualities). Furthermore, under certain conditions the various branes can dynamically transform into each other as well as into fundamental strings. As a result, the physics of p-branes has played an increasingly important role in the understanding of M-theory as a whole.

It turns out that p-branes are far more complicated objects than are strings. One therefore uses a variety of techniques to study them, each of which applies in a different region of parameter space. These include string perturbation theory, brane effective actions, and supergravity techniques. By splicing together these pictures, researchers obtain new insights into brane dynamics and the theory in which they live.

At Syracuse, such studies are pursued mainly using supergravity physics and the related brane effective actions. The basic idea here is that the branes of M-theory are related to higher-dimensional generalizations of black holes. A review by Don Marolf provides an introduction for students with a background in general relativity.

The Maldacena Conjecture (AdS/CFT)

Perhaps the most shocking outgrowth of the physics of branes has been the Maldacena conjecture. This conjecture states that M-theory subject to particular boundary conditions is in fact equivalent to some supersymmetric Yang-Mills (i.e., non-gravitational!) theory on a manifold of smaller dimension! One example is the so-called AdS/CFT correspondence, in which string theory with boundary conditions matching the ten-dimensional manifold given by the product of 4+1 Anti-DeSitter space and a five-sphere (AdS5 x S5) is conjectured to be equivalent to 3+1-dimensional super Yang-Mills theory, a four-dimensional conformal field theory (CFT). This surprising idea follows from certain arguments involving taking the low energy limit of D-brane physics from both the spacetime (gravitating) point of view and from the point of view of string perturbation theory. Unfortunately, no version of this conjecture is currently known which would apply to asymptotically flat spacetimes (such as Minkowski space)..

Although the conjecture has not yet been proven, an impressive variety of supporting evidence has been obtained. These range from the classification of linearized perturbations to calculations of black hole entropy (see below). Another piece of such evidence stems from the studies of gravitating branes mentioned above. Marolf and Sumati Surya (a past Syracuse student, now at UBC) used supergravity techniques to uncover certain links between brane physics and black hole no-hair theorems. This work was then extended by Marolf and Amanda Peet (Toronto) and the Maldacena conjecture was used to suggest a `dual version' of the effect in the super Yang-Mills quantum field theory description. By showing that quantitative information governing the no-hair phenomenon was reproduced by the appropriate quantum field theory calculation, they added a new piece of evidence in support of the Maldacena conjecture and refined the `dictionary' that translates between the gravitating and non-gravitating sides of the correspondence.

The correspondence can also be used in the other direction. As an example, Marolf and Peet turned their arguments around to predict certain gravitational features of branes. Supporting evidence for these predictions was then found by Marolf, Andres Gomberoff (then a postdoc at Syracuse, now at CECS), David Kastor (U. Mass) and Jennie Traschen (U. Mass). A more detailed analysis using numerical techniques is now being pursued in conjunction with Pablo Laguna (Penn State).

However, this phenomenon may yet have more more to teach us. Marolf and Pedro Silva are exploring this possibility by investigating the relationship between the above no-hair results and non-abelian D-brane effective actions, which is another story in itself.

Field Theory and Non-Commutative Geometry

Recently, it has been shown that field theories on so-called non-commutative spaces also play a role in M-theory and shed light on interesting questions of brane dynamics. A non-commutative geometry is an algebraic generalization of a manifold (with metric) in which the coordinates do not commute. As an example, one could roughly refer to a quantum mechanical Hilbert space as a non-commutative phase space. At Syracuse, the study of non-commutative geometry has been pursued for some time by A. P. Balachandran and by Kamesh Wali. Be sure to read the corresponding entry under Elementary Particles and Fields for a description of this work.

Black Holes and Quantum Mechanics in M-theory

Black holes have long been a focal point for studies of quantum gravity. In part, this stems from dimensional analysis which suggests that the fundamental physics of quantum gravity takes place at the Plank scale, roughly 10-35 meters. The fact that quantum fluctuations in vacuum energy can create black holes at this scale suggests that the fundamental structure may be a `soup of virtual black holes,' sometimes known as `spacetime foam.' The other reason for the focus on black holes is the intriguing phenomenon of Hawking radiation, first uncovered by Stephen Hawking in the early 1970's. Although it is not possible for any energy to escape from a black hole in classical physics, quantum effects cause black holes to radiate like black bodies. The corresponding temperature is tiny for everyday black holes, but is large for tiny Plank scale Schwarzschild black holes. Since black holes have a temperature, they also have an entropy, which turns out to be enormous but finite and an intense point of discussion. The tension between the classical notion of causality (which is, after all, what determines that nothing can escape from a black hole) and Hawking radiation also suggests that quantum gravity effects may cause a fundamental shift in our understanding of space and time. The study of such issues sometimes goes under the heading of `the information paradox,' which refers to the issue of whether information that enters a black hole can in fact leave again through quantum processes.

String (or M-) theory provides a number of tools that can be used to study the quantum physics of black holes. (Be sure to also read the discussion of black holes and quantum mechanics under Classical and Quantum Gravity.) One of the most powerful has been the use of D-brane techniques. D-branes are non-perturbative objects around which string perturbation theory can still describe physics. In this context they are well known as places where strings can end. Placing enough D-branes together can create a black hole. As first described by Andrew Strominger and Cumrun Vafa, string techniques then predict certain properties of this black hole. In particular, such methods have been used to successfully calculate both Hawking radiation from the hole and the entropy of these black holes. These are the only known techniques through which one can precisely predict the entropy of a black hole by counting microscopic states. Interestingly, such calculations are done in a regime in which no horizon exists -- supersymmetry is used to extrapolate the result to honest black holes. As a result, many fundamental questions remain and are the subject of on-going research. Marolf has participated [1,2,3] in the use of D-brane techniques to probe black hole entropy and information and continues to address such issues, e.g. recent work with

Jorma Louko (Nottingham) and Simon Ross (Durham).

A related topic is the idea of `holography,' which suggests that a fundamental description of an n+1 dimensional spacetime may in fact be through an n-dimensional theory (or, more properly, and (n-1)+1 dimensional theory). This idea was originally suggested by Lenny Susskind, Willy Fischler, Gerard t'Hooft, and others motivated by the fact that the entropy of black holes scales with their surface area instead of their volume. Assuming that the Maldacena conjecture is correct, it provides a striking implementation of this idea.

A particular version of holography is known as the Bousso conjecture. While less sweeping (and less precise) than the Maldacena conjecture, it has the advantage that it can in fact apply to general spacetimes which need not satisfy special boundary conditions. A rough statement of Bousso's conjecture is that the entropy flux through any null surface is bounded by the area of this null surface. In a recent paper, Marolf, Eanna Flanagan (Cornell), and Robert Wald (Chicago) were able to prove that this bound in fact follows from conventional Einstein gravity in the appropriate semi-classical setting.

String Cosmology

Mark Bowick, Mark Trodden, Joel Rozowsky and Salah Nasri are studying elements of superstring cosmology. In particular they are interested in the issue of the dimensionality of spacetime.

Nonperturbative effects from geometry

An important feature of M-theory is that, at least in certain regimes, it is properly described as an eleven-dimensional theory. This is in contrast to the original string theory which lives in ten dimensions. These descriptions of the theory are related through the process of Kaluza-Klein reduction, where a higher dimensional theory can be made to seem like a lower dimensional theory containing extra fields. The ten dimensional description arises when one of the eleven dimensions is a circle whose size is small enough to be ignored.

The original formulation of string theory in terms of the scattering of quantum strings makes use of a small parameter known as the string coupling, g. This description is inherently tied to a perturbative expansion in powers of g. Now, the string coupling turns out to be related to the size of the tiny circle that constitutes the eleventh dimension. Small g arises for small circles while large g arises for large circles.

For large g, one may consider situations in which quantum effects are small so that one can use classical eleven-dimensional gravity to accurately describe the physics. While the description in terms of string scattering is inherently perturbative, eleven-dimensional gravity is not. Thus, one can use properties of eleven-dimensional gravity to obtain non-perturbative information about M-theory. In some cases, one can use supersymmetry to argue that classically derived conclusions also remain valid when quantum mechanics is taken into account.

An excellent example of this kind of result is the Kaluza-Klein monopole, discovered by Rafael Sorkin long before the days of M-theory. This is a stable solution to the 4+1-dimensional Einstein equations whose 3+1-dimensional description is as a magnetic monopole in gravity coupled to an electromagnetic field (and a scalar field). While magnetic monopoles are singular, in this case the singularity is merely an artifact of the 3+1-dimensional description. The 4+1 description is a perfectly smooth spacetime. Thus, the higher dimensional geometry implies that such a theory does in fact contain magnetic monopoles.

Kaluza-Klein monopoles (generalized to 9+1 and 10+1 dimensions) continue to be of importance in M-theory, and in fact they have the same status as the p-branes described above. The monopoles are related to various branes by the duality symmetries of M-theory, and in fact one D-brane can described as a Kaluza-Klein monopole in eleven dimensions. An example of how these monopoles can be used to derive non-perturbative effects in string theory can be found in a recent paper by Marolf which uses their eleven-dimensional geometry to resolve certain issues involving charge quantization. The monopole geometry makes a single brane (known as a M2-brane) in eleven dimensions appear as a pair of D-branes in ten dimensions. Not surprisingly, these two branes must always remain attached to each other. This leads to a phenomenon in which certain external fields cause D-branes to be confined in pairs. Further studies of Kaluza-Klein monopoles and other aspects of eleven-dimensional geometry are certain to uncover additional effects that are invisible to string perturbation theory.